Given a domain D with elements $d \in D$, and a reification function δ:

$$incomp(Q) \stackrel{\triangle}{=} \forall\, a, b \in Q.\, (a \neq b \implies a \sqcup b = \top)$$

$$\boxed{\langle S;\, e\rangle \hookrightarrow \langle S';\, e'\rangle}$$

(where $\langle S';\, e'\rangle \neq$ **error**)

E-REFL
$$\frac{}{\langle S;\, e\rangle \hookrightarrow \langle S;\, e\rangle}$$

E-PARAPP
$$\frac{\langle S;\, e_1\rangle \hookrightarrow \langle S_1;\, e_1'\rangle \qquad \langle S;\, e_2\rangle \hookrightarrow \langle S_2;\, e_2'\rangle \qquad \langle S_1^r;\, e_1'^r\rangle = rename(\langle S_1;\, e_1'\rangle, S_2, S) \qquad S_1^r \sqcup_S S_2 \neq \top_S}{\langle S;\, e_1\ e_2\rangle \hookrightarrow \langle S_1^r \sqcup_S S_2;\, e_1'^r\ e_2'\rangle}$$

E-PUT-1
$$\frac{\langle S;\, e_1\rangle \hookrightarrow \langle S_1;\, e_1'\rangle}{\langle S;\, \mathsf{put}\ e_1\ e_2\rangle \hookrightarrow \langle S_1;\, \mathsf{put}\ e_1'\ e_2\rangle}$$

E-PUT-2
$$\frac{\langle S;\, e_2\rangle \hookrightarrow \langle S_2;\, e_2'\rangle}{\langle S;\, \mathsf{put}\ e_1\ e_2\rangle \hookrightarrow \langle S_2;\, \mathsf{put}\ e_1\ e_2'\rangle}$$

E-PUTVAL
$$\frac{S(l) = d_2 \qquad d_1 \in D \qquad d_1 \sqcup d_2 \neq \top}{\langle S;\, \mathsf{put}\ l\ \{d_1\}\rangle \hookrightarrow \langle S[l \mapsto d_1 \sqcup d_2];\, \{\}\rangle}$$

E-GET-1
$$\frac{\langle S;\, e_1\rangle \hookrightarrow \langle S_1;\, e_1'\rangle}{\langle S;\, \mathsf{get}\ e_1\ e_2\rangle \hookrightarrow \langle S_1;\, \mathsf{get}\ e_1'\ e_2\rangle}$$

E-GET-2
$$\frac{\langle S;\, e_2\rangle \hookrightarrow \langle S_2;\, e_2'\rangle}{\langle S;\, \mathsf{get}\ e_1\ e_2\rangle \hookrightarrow \langle S_2;\, \mathsf{get}\ e_1\ e_2'\rangle}$$

E-GETVAL
$$\frac{S(l) = d_2 \qquad incomp(Q) \qquad Q \subseteq D \qquad d_1 \in Q \qquad d_1 \sqsubseteq d_2}{\langle S;\, \mathsf{get}\ l\ Q\rangle \hookrightarrow \langle S;\, \{d_1\}\rangle}$$

E-REIFY
$$\frac{\langle S;\, e\rangle \hookrightarrow \langle S';\, e'\rangle}{\langle S;\, \mathsf{reify}\ e\rangle \hookrightarrow \langle S';\, \mathsf{reify}\ e'\rangle}$$

E-REIFYVAL
$$\frac{}{\langle S;\, \mathsf{reify}\ Q\rangle \hookrightarrow \langle S;\, \delta(Q)\rangle}$$

E-BETA
$$\frac{}{\langle S;\, (\lambda x.\, e)\ v\rangle \hookrightarrow \langle S;\, e[x := v]\rangle}$$

E-NEW
$$\frac{}{\langle S;\, \mathsf{new}\rangle \hookrightarrow \langle S[l \mapsto \bot];\, l\rangle}\ (l \notin dom(S))$$

$$\boxed{\langle S;\, e\rangle \hookrightarrow \textbf{error}}$$

E-REFLERR
$$\frac{}{\textbf{error} \hookrightarrow \textbf{error}}$$

E-PARAPPERR
$$\frac{\langle S;\, e_1\rangle \hookrightarrow \langle S_1;\, e_1'\rangle \qquad \langle S;\, e_2\rangle \hookrightarrow \langle S_2;\, e_2'\rangle \qquad \langle S_1^r;\, e_1'^r\rangle = rename(\langle S_1;\, e_1'\rangle, S_2, S) \qquad S_1^r \sqcup_S S_2 = \top_S}{\langle S;\, e_1\ e_2\rangle \hookrightarrow \textbf{error}}$$

E-APPERR-1
$$\frac{\langle S;\, e_1\rangle \hookrightarrow \textbf{error}}{\langle S;\, e_1\ e_2\rangle \hookrightarrow \textbf{error}}$$

E-APPERR-2
$$\frac{\langle S;\, e_2\rangle \hookrightarrow \textbf{error}}{\langle S;\, e_1\ e_2\rangle \hookrightarrow \textbf{error}}$$

E-PUTERR-1
$$\frac{\langle S;\, e_1\rangle \hookrightarrow \textbf{error}}{\langle S;\, \mathsf{put}\ e_1\ e_2\rangle \hookrightarrow \textbf{error}}$$

E-PUTERR-2
$$\frac{\langle S;\, e_2\rangle \hookrightarrow \textbf{error}}{\langle S;\, \mathsf{put}\ e_1\ e_2\rangle \hookrightarrow \textbf{error}}$$

E-PUTVALERR
$$\frac{S(l) = d_2 \qquad d_1 \in D \qquad d_1 \sqcup d_2 = \top}{\langle S;\, \mathsf{put}\ l\ \{d_1\}\rangle \hookrightarrow \textbf{error}}$$

E-GETERR-1
$$\frac{\langle S;\, e_1\rangle \hookrightarrow \textbf{error}}{\langle S;\, \mathsf{get}\ e_1\ e_2\rangle \hookrightarrow \textbf{error}}$$

E-GETERR-2
$$\frac{\langle S;\, e_2\rangle \hookrightarrow \textbf{error}}{\langle S;\, \mathsf{get}\ e_1\ e_2\rangle \hookrightarrow \textbf{error}}$$

E-REIFYERR
$$\frac{\langle S;\, e\rangle \hookrightarrow \textbf{error}}{\langle S;\, \mathsf{reify}\ e\rangle \hookrightarrow \textbf{error}}$$

Figure 4. An operational semantics for λ_{par}.